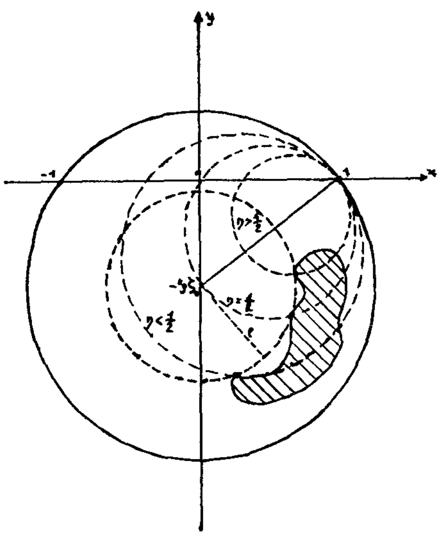

Fig. 1.

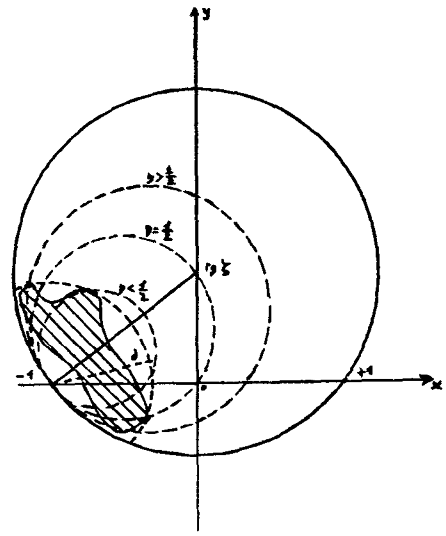

Fig. 2.

veloped gave the solution of the problem directly, without any cut and try.

As the question may present some technical interest, copy of our manuscript follows.

SÉANCE DU 30 JANVIER 1961, p. 689.
ÉLECTRONIQUE.—*Sur le problème de l'adaptation.* Note de MM. Louis Castagnetto et Jean-Claude Matheau, présentée par M. Léopold Escande.

On détermine à l'aide du diagramme de Smith, les conditions d'adaptation optimale d'une charge à un élément actif lorsque les domaines de variation des impédances sont préalablement imposés.

Soient $R_0+jX_0=|Z_0|e^{j\zeta_0}$ l'impédance du générateur, $R+jX=|Z|e^{j\zeta}$ celle de la charge et $S=x+jy$ son coefficient de réflexion.

$|E|e^{j\phi}$ étant l'amplitude complexe de la force électromotrice du générateur, la puissance complexe absorbée par la charge s'écrit

$$H = P + jQ = \frac{|E|^2}{8Z_0^*}(1+S)(1-S^*)$$
$$= \frac{|E|^2}{8Z^*}|1+S|^2.$$

Alors:

$$P = \frac{|E|^2}{8R_0}(1-\rho^2\cos^2\zeta_0) = \frac{|E|^2}{8R}d^2\cos^2\zeta,$$

où ρ est, dans le plan du coefficient de réflexion, la distance du point S au point $(0, -tg\zeta_0)$ et d la distance du point S au point $(-1, 0)$.

L'impédance du générateur étant donnée ainsi que le domaine de variation de S la puissance maximale absorbée par la charge sera obtenue aux points S où ρ atteint sa borne inférieure. On peut alors choisir, parmi l'ensemble des points donnant la valeur maximale, ceux qui correspondent au rendement maximal.

Le rendement s'écrit

$$\eta = \frac{R}{R+R_0}.$$

Les courbes à rendement constant (R constant dans le plan des impédances) se transforment par l'homographie donnant S, en un faisceau de cercles tangents au point (1, 0) centrés sur le segment $[(0, -\text{tg}\,\zeta_0), (1, 0)]$.

L'ensemble de ces cercles figurant dans le diagramme de Smith, la détermination de l'adaptation optimale est alors immédiate (Fig. 1).

D'une façon similaire, si l'impédance Z est donnée ainsi que le domaine de variation de S, l'impédance variable étant maintenant Z_0, la puissance maximale absorbée par la charge sera obtenue aux points S où d atteint sa borne supérieure. Les courbes à rendement constant forment un faisceau de cercles tangents au point $(-1, 0)$ et centrés sur le segment $[(-1, 0), (0, \text{tg}\,\zeta)]$.

Le choix du point donnant la puissance maximale avec le rendement maximal se fait encore immédiatement (Fig. 2).

Si les domaines de variation sont donnés dans le plan Z il suffit de passer au plan S pour résoudre le problème.

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Author's Comment[2]

Castagnetto and Matheau essentially have solved the same problem treated in my correspondence.[1] The main difference is that the entire description is carried out in the reflection coefficient plane rather than the impedance plane. They take the question one step further by exchanging the role of source and load impedance. The equations are correct and convenient to use on the Smith Chart.

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Discontinuities Appear in the Repeller Current of a Reflex Klystron Detector*

It is known that reflex klystrons are usable as microwave detectors.[1–4] When the repeller current was plotted against acceleration grid (which is anode) voltage, Koctienko, Deviatkov, and Lebed[5] found discontinuities. Whitford[4] did not find these discontinuities for the 726A reflex klystron. Koctienko, *et al.*, explained that the discontinuities were attributed to the appearance and disappearance of the virtual cathode when the klystron began or stopped oscillation. Whitford explained that the beam current density of the 726A reflex klystron was sufficiently high to maintain the virtual cathode at all times.

The author found discontinuities in the repeller current as shown by a broken line in Fig. 1 when the 2K25 reflex klystron was used as a microwave detector. These discontinuities were explained in this case from the viewpoint of electronic regenerative action instead of by the appearance and disappearance of the virtual cathode. In this analysis, for simplicity, a plane parallel electrode reflex klystron as shown in Fig. 2 was assumed. The repeller was connected to the cathode and the virtual cathode was assumed to always exist in front of the actual cathode. Another virtual cathode in front of the repeller was not considered at this time because of low repeller current on the order of microamperes. Initial energy δ, which consisted of Φ, average thermal energy of electrons converted into kinetic energy at the emission and energy of the electric field intensity E_0 at the virtual cathode surface to pull the electron out of the potential dip to the virtual cathode surface where the potential was zero was assumed. The repeller current due to the acceleration grid voltage V_g and the initial energy is given by

$$i_{r0} = C\frac{\xi I_0}{U_{g0}}U_{r0} \qquad (1)$$

where

C is a ratio of total charge at the repeller to the total charge at the grid, ξ is the beam transmission factor of the grids, I_0 is the grid current, and

$$I_0 \approx K_0 V_g^{3/2}. \qquad (2)$$

This approximate relation was verified by the experiment. K_0 is a proportionality constant between the grid current and the grid voltage. U_{g0} is the velocity of electron

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[1] A. E. Harrison, "Klystron Tubes," McGraw-Hill Book Co., Inc., New York, N. Y., 1st ed.; 1947.

[2] S. A. Kornelov and O. N. Kazbekova, "Detection in the cathode circuit of an under excited reflex klystron," *Radiotekh. i Elektron.*, vol. 4, pp. 475–481; March, 1959.

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[5] A. I. Koctienko, M. N. Deviatkov, and A. A. Lebed, "On the application of virtual cathodes for the detection of microwave signals," *Radiotekh. i Elektron.*, vol. 4, pp. 482–488; March, 1959.

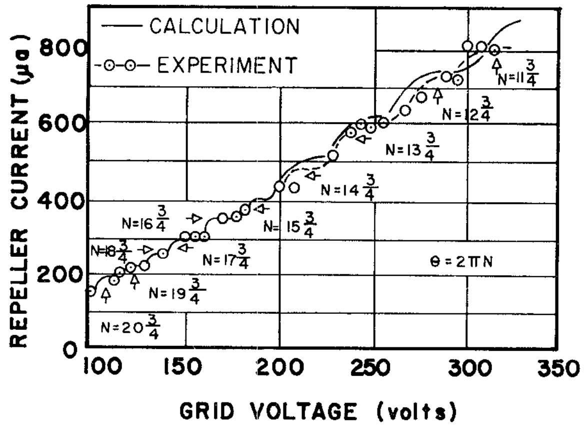

Fig. 1—Repeller current due to microwave signals of the 2K25 reflex klystron with repeller connected to cathode.

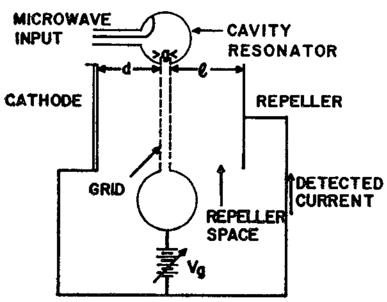

Fig. 2—Plane parallel electrode reflex klystron detector.

at the grid and

$$U_{g0} = \sqrt{\frac{2eV_g}{m} + \frac{2\delta}{m}}$$

$$= \sqrt{\frac{2eV_g}{m} + \frac{2}{m}(\Phi + eE_0S)}. \quad (3)$$

In this equation

e = the electronic charge,
m = the electronic mass,
δ = the initial energy,
S = an equivalent distance between the potential dip and the surface of the virtual cathode—assumed, for simplicity, to be constant in this case, and
E_0 = the virtual cathode surface electric field intensity.

This is given by integrating Poisson's equation

$$E_0 = \sqrt{\frac{1}{d^2} - \frac{4K_0}{\epsilon_0 A_g}\sqrt{\frac{m}{2e}}\, V_g} \quad (4)$$

where

ϵ_0 = the dielectric constant of vacuum,
d = the distance between the cathode and the grid, and
A_g = the effective area of the grids.

In (1), U_{r0} is the velocity of electrons at the repeller and this is given by

$$U_{r0} = \sqrt{U_{g0}^2 - \frac{2eV_g}{m}} \quad (5)$$

where U_{g0} is given by (3).

Substituting (2)–(5) into (1),

$$i_{r0} = C\xi K_0 V_g^{3/2}\sqrt{\frac{A_1V_g + \Phi}{A_2V_g + \Phi}} \quad (6)$$

where

$$A_1 \equiv eS\sqrt{\frac{1}{d^2} - \frac{4K_0}{\epsilon_0 A_g}\sqrt{\frac{m}{2e}}} \equiv A_2 - eS. \quad (7)$$

The average repeller current, due to microwaves $\bar{i}_{rm}$, is obtained using the following assumptions. Microwaves are assumed to contribute repeller current directly only when the microwaves are in the accelerating phases of repeller bound electrons at the grids. The electrons returning to the grids are assumed to contribute to the regenerative amplification providing the grid voltage is properly adjusted. It is also assumed that there will be no contribution of microwaves to the repeller current when the regenerative amplification was not being performed because of incorrect grid voltage settings.

The average repeller current then due to microwaves is

$$\bar{i}_{rm} = \frac{1}{2\pi}\int_{2n\pi}^{(2n+1)\pi} i_{rm}\, d(\omega t)$$

$$= \frac{1}{2\pi}\int_{2n\pi}^{(2n+1)\pi} C\frac{\xi I_0}{U_{g0}}\cdot U_{g0}\sqrt{2K\beta}\sin\omega t\; d(\omega t)$$

$$= \frac{1.198C\xi I_0}{\pi}\sqrt{2K\beta} \quad (8)$$

where

$$K \equiv \frac{V_0}{2V_g} \equiv \frac{v_i}{2V_g\sqrt{1 - \frac{R_L\beta^2\theta\xi I_0\cos\phi}{2V_g}}}, \quad (9)$$

where

V_0 = the amplified microwave voltage across the grid by regenerative amplification.
ϕ = a phase angle between induced microwave current in the circuit at the grids and microwave voltage across the grids,

$$\cos\phi = \cos\left\{4\omega l\sqrt{\frac{m}{2e}}\, V_g^{-1/2} - 2\pi(n - \tfrac{1}{4})\right\}. \quad (10)$$

n = a positive integer.
l = the distance between the grids and the repeller.
ω = the operating angular frequency, $\omega = 2\pi f$.
v_i = the magnitude of input microwave voltage across the grids.
R_L = the parallel resistance of the circuit across the grids.
θ = the electron transit angle in the repeller space,

$$\theta = 8\pi fl\sqrt{\frac{m}{2e}}\, V_g^{-1/2}. \quad (11)$$

β = the beam coupling coefficient,

$$\beta = \frac{2}{\omega g}\sqrt{\frac{2e}{m}}\, V_g^{1/2}\sin\frac{g\omega}{2}\sqrt{\frac{m}{2e}}\, V_g^{-1/2}. \quad (12)$$

g = the gap length of the grids.

Substituting (2), (9) and (12) in (8) with the assumptions stated before (8),

$$\left.\begin{aligned} \bar{i}_{rm} &= A_3\frac{V_g^{5/4}\sin^{1/2}(A_4V_g^{-1/2})}{\sqrt{1 - A_5\cos\phi\cdot V_g\sin^2(A_4V_g^{-1/2})}} && \text{when } \cos\phi > 0 \\ \bar{i}_{rm} &\approx 0 && \text{when } \cos\phi < 0 \end{aligned}\right\} \quad (13)$$

where

$$A_3 = \frac{1.198C\xi K_0}{\pi g\omega}\sqrt{\frac{4ev_i}{m}}, \quad (14)$$

$$A_4 = \frac{g\omega}{2}\sqrt{\frac{m}{2e}}, \quad (15)$$

$$A_5 = \frac{16C\xi K_0R_Ll}{g^2\omega}\sqrt{\frac{m}{2e}}. \quad (16)$$

Total repeller current is then

$$i_r = i_{r0} + \bar{i}_{rm}. \quad (17)$$

This equation is shown by a solid line in Fig. 1. Parameters used to calculate this curve are listed in Table I. Steps in the repeller current are clearly shown in this theoretical curve. The electron transit cycles in the repeller space,

$$N = \frac{\theta}{2\pi} = \frac{\phi - (n - \frac{1}{4})}{2\pi},$$

are also shown in Fig. 1. As seen in this figure, steps appearing in the repeller current curve coincide approximately with the mode of regenerative amplification which is represented by the electron transit cycles N.

TABLE I
PARAMETERS FOR THE 2K25 REFLEX KLYSTRON DETECTOR

Parameters	Numerical Values
$C\xi K_0$	0.16 μa/volt
K_0	0.0042 ma/volt$^{3/2}$
Φ	1.602 ×10^{-19} joule
A_1	6.635 ×10^{-20} coulomb
A_2	7.29 ×10^{-20} coulomb
A_3	5.8 ×10^{-2} ampere/volt$^{5/4}$
A_4	29.95 rad-volt$^{1/2}$
A_5	1.171 ×10^{-2} volt^{-1}
f	9200 Mc
l	3.44 ×10^{-3} m

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